Operator matrices and the problem on small movements of a viscous fluid in an open vessel

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There were investigated classical initial boundary value problem on small movements of a viscous fluid in an arbitrary open vessel (famous S. G. Krein's problem):

$$\begin{gathered}\frac{\partial \mathbf{u}}{\partial t} = -\rho^{-1}\nabla p + \nu\Delta\mathbf{u} + \mathbf{f}(t,x), \quad \operatorname{div}\mathbf{u} = 0 \quad (\text{ in } \Omega\,),\\ \mathbf{u} = 0 \quad (\text{ on } S\,), \quad \frac{\partial \zeta}{\partial t} = u_n := \mathbf{u}\cdot\mathbf{n} \quad (\text{ on } \Gamma\,), \quad \int_\Gamma \zeta\, d\Gamma = 0,\\ \nu\left(\frac{\partial u_i}{\partial x_3} + \frac{\partial u_3}{\partial x_i}\right) = 0 \quad (\, i = 1,2;\ \text{on } \Gamma\,), \quad -p + 2\rho\nu\frac{\partial u_3}{\partial x_3} = -\rho g\zeta \quad (\text{on}\,\Gamma),\\ \mathbf{u}(0,x) = \mathbf{u}^0(x),\ x\in\Omega, \quad \zeta(0,\hat{x}) = \zeta^0(\hat{x}),\ \hat{x}\in\Gamma.\end{gathered} \tag{1}$$

Problem (1) by using approaches developed in [1]—[2] is reduced to Cauchy problem

$$\frac{dy}{dt} + \mathcal{A}y = f(t), \quad y(0) = y^0, \tag{2}$$

where the function $y(t) = (\mathbf{u};\zeta)^t$ with values in some Hilbert space $\mathcal{H}$ describes the evolution of the velocity field $\mathbf{u}(t,x)$ of a fluid and the displacement $\zeta(t,x_1,x_2)$ of moving free surface from equilibrium one Γ.

In the lecture we prove the theorem on strong solvability of problem (1) and investigate spectral properties of corresponding problem on normal oscillations on the base of Cauchy problem (2), where operator matrix $\mathcal{A}$ is maximal accretive operator acting in $\mathcal{H}$.

References

[1] F. V. Atkinson, H. Langer, R. Mennicken, A. A. Shkalikov. The essential spectrum of some matrix operators, Math. Nachr. 167 (1994), 5–20.

[2] T. Ya. Azizov, N. D. Kopachevsky, L. D. Orlova. Evoliutsionnije i spectral'nije zadachi, porozhdionnije problemoy malih dvizheniy viazkouprugoy zhidkosti, Trudy Sankt–Peterburgskogo matemat. obschectva, Tom 6 (1998), 5–33 (in Russian).