Abstract Green's formula and transmission problems

N. Kopachevsky, P. Starkov

We study transmission problem

$$u - \Delta u + \lambda u = 0, \ (in\ \Omega \subset \mathbb{R}^m), \quad \frac{\partial u}{\partial n} = \mu u \quad (on\ \partial\Omega), \tag{1}$$

for Helmholtz equation. Here $\lambda,\ \mu \in \mathbb{C}$, λ is a fixed parameter, μ is a spectral one. Using operator approach (see, for instance, Kopachevsky N. D., Krein S. G., Ngo Zuy Can "Operatornie metodi v lineinoi gidrodinamike" M: Nauka - 1989, 413p.) we reduce problem (1) to investigation on the operator pencil

$$M(\mu) = (I + \lambda A - \mu B), \quad 0 < A \in \mathcal{S}_\infty, \quad 0 \leq B \in \mathcal{S}_\infty. \tag{2}$$

We use the same approach for studying so-called exterior, the first and the second transmission problems. All of these problems are formulated as spectral ones for operator pencil (2).

The above mentioned approach can be used for investigation abstract transmission problem

$$\mathcal{L}u = \lambda u, \quad \partial u = \mu \gamma u,$$

on the base of abstract Green's formula

$$< \mathcal{L}u, v >_E = (u, v)_F - < \partial u, \gamma v >_G$$

under some assumptions on Hilbert spaces E, F and G.

We give applications of these general approach to transmission problems for Navie-Stokes equations, elasticity equations and some classes of elliptic problems.

N. Kopachevsky kopachevsky@tnu.crimea.ua
P. Starkov pstarksci@aport.ru
Math. analysis chair, Taurida National
V. Vernadsky University, ul. Yaltinskaya, 4,
Simferopol, Crimea, 95007, Ukraine.