On Some Classes of Complete Volterra Integro-Differential Equations in Hilbert Space

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The report deals with the Cauchy problem for linear Volterra first order integro-differential equations

$$\frac{du}{dt} + Fu + \sum_{k=1}^{m} \int_0^t G_k(t,s)A_k u(s)ds = f(t), \quad u(0) = u_0, \tag{1}$$

and the second order equations

$$\begin{gathered}\frac{d^2u}{dt^2} + (F+iG)\frac{du}{dt} + Bu + \sum_{k=1}^{m} \int_0^t G_k(t,s)A_k u(s)ds = f(t), \\ u(0) = u_0,\ u'(0) = u_1.\end{gathered} \tag{2}$$

Here, $u = u(t)$ is an unknown function of the variable t with values in a Hilbert space $\mathcal{H}$, $f(t)$ is a given function, $F \gg 0$, $G = G^*$, $B \gg 0$ and A_k, $k = \overline{1,m}$, are the operator coefficients, which are, generally speaking, unbounded operators, $G_k(t,s)$ is an operator function of variables t,s with values in $\mathcal{L}(\mathcal{H})$.

It is supposed that the domains of the operators F, G, B and A_k are comparable, and the following three classes of the differential (if $A_k = 0$) and the integro-differential equations are considered: 1°. The strongly damped dynamic systems: $\mathcal{D}(F) \subset \mathcal{D}(B)$, $\mathcal{D}(A_k) \supset \mathcal{D}(F)$. 2°. The weakly damped dynamic systems: $\mathcal{D}(F) \subset \mathcal{D}(B^{1/2})$, $\mathcal{D}(A_k) \supset \mathcal{D}(B^{1/2})$. 3°. The average damped dynamic systems: $\mathcal{D}(B) \subset \mathcal{D}(F) \subset \mathcal{D}(B^{1/2})$, $\mathcal{D}(A_k) \supset \mathcal{D}(B^{1/2})$.

For these classes of equations and some of their generalizations the theorems on the existence of strong solutions on an arbitrary time interval $[0,T]$ are proved. For this the theory of semigroups operators, the theory of operator block matrices and other functional analysis methods are used.

The examples of problems (1) and (2) that are associated with small motions of a viscoelastic fluid and relaxing fluid, as well as some problems for the complete second order differential equations that are appeared in fluid dynamics are given.

[1] N.D. Kopachevsky, R. Mennicken, Yu.S. Pashkova, C. Tretter. *Complete Second Order Linear Differential Operator Equations in Hilbert Space and Applications in Hydrodynamics*. Transactions of the AMS, **356**, **12** (2004), pp. 1737–1766.

[2] N.D. Kopachevsky. *Integro-Differential Equations of Volterra in Hilbert Space*. Simferopol, 2012. – 152 p.