

ANATOLY DMITRIEVICH MYSHKIS  
(ON THE 90-TH ANNIVERSARY)

N.D. KOPACHEVSKY\*, L.E. ROSOVSKII† AND  
A.L. SKUBACHEVSKII‡

**Childhood and school years.** Anatoly Dmitrievich Myshkis was both an outstanding mathematician theorist and an applied mathematician as well as a wonderful teacher.

A.D. Myshkis was born on the 13th of April 1920 in the town of Spassk of Ryazan Region. When he was a year and a half, the family moved to Kharkiv and had lived there until the fall of 1932. His father Dmitrii Semyonovich Ermakov, wounded in the First World War, was a worker at the Putilov plant. Anatoly Dmitrievich's mother, Haya Samuilovna Myshkis, knew the German and the French languages and taught at the Communist International School.

In 1932, the father of Anatoly Dmitrievich was transferred to another job in Moscow. Thereby A.D. began studying in the sixth form at the forest boarding school in Sokol'niki. He was then admitted to the seventh form of exemplary school number 25 of Moscow in September 1933. The children of Stalin, Molotov, Kuybyshev and some other well-known political figures went to that school at different times.

Since September 1935, among the classmates of A.D. were L. Ovsyanikov, who became an academician afterwards, and T. Schneider. Three of them were much higher in mathematics than the rest of the class. Opening of the mathematical circle for schoolchildren at the mechanical-mathematical faculty (mechmath) of the Moscow State University (MSU) became an extremely important event in the life of the young men. The first lesson at the

---

\* Department of Mathematical Analysis, Taurida National V.I. Vernadsky University, 4 Academician Vernadsky Ave., Simferopol, Ukraine

† Department of Differential Equations and Mathematical Physics, Peoples' Friendship University of Russia, 6 Mikhlukho-Maklaya Str., Moscow, Russia

‡ Department of Differential Equations and Mathematical Physics, Peoples' Friendship University of Russia, 6 Mikhlukho-Maklaya Str., Moscow, Russia

mathematical circle, where A.D. came with his friends, was conducted by I.M. Gel'fand, who was also very young. From that moment, I.M. Gelfand had played the role of preceptor for A.D.

A.D. familiarized himself with the notions of higher mathematics already in the eighth form. So he studied the last years at school without making big efforts.

**Studying at the mechmath of the MSU.** After finishing school with excellent marks all of the three classmates were admitted to the mechmath without examinations. Here again A.D. met I.M. Gel'fand among the other outstanding mathematicians and teachers. I.M. Gel'fand conducted practicals on mathematical analysis and A.D. received a special care from him up to the fourth year at the university. It should be noted that A.D. regarded the following prominent scientists as his teachers: I.M. Gel'fand, I.G. Petrovskii, and physicist theorist Ya.B. Zel'dovich.

A.D.'s lecturer on differential equations on the second year at the university was I.G. Petrovskii. During the third year S.L. Sobolev lectured on mathematical physics, while the practical training was conducted by I.G. Petrovskii. In the sequel, A.D. attended the lectures on complex analysis by I.I. Privalov, the lectures on probability theory by A.N. Kolmogorov, and the lectures on functional analysis by I.M. Gel'fand. A.D. however preferred reading textbooks and monographs devoting all of his spare time to mathematics. Besides that, A.D. practised music and spent about three hours a day on the exercises on the violin.

According to A.D., he read the manuscript of Petrovskii's lectures on differential equations at that time and, having found several inaccuracies, told I.G. Petrovskii his reasons, with which I.G. immediately agreed. As the course of lectures came later off the press, the author acknowledged A.D. in the preface. This kind of attitude had a strong effect on the young student.

**The War period.** The war began when A.D. finished his fourth year at the MSU. All of the students of the mechmath were thrown to the vicinities of Moscow for digging of trenches and anti-tank ditches. Those who finished the fourth year were returned to Moscow in the beginning of September 1941, as the front came close to the city. They were then sent to Sverdlovsk to study at the N.E. Zhukovsky Military-Air Engineering Academy (MAEA).

A.D. Myshkis succeeded to take an external degree at the MSU, and the three-year study at the aircraft armament faculty of the MAEA began. Those years were very difficult. Intensive studies combined with the cold, hunger, harassing physical training and drill resulted in tiredness and torpor.

A.D. met I.M. Gel'fand again in September 1942 in Sverdlovsk where the

mechmath of the MSU was moved to from Ashkhabad. Following Gel'fand's advice, he decided to enter the postgraduate study at the mechmath under the supervision of I.G. Petrovskii. By that time A.D. graduated from the MAEA with honours. The title of his thesis at the department of air shooting was "Rifle and cannon armament of the multipurpose aircraft TU-2". In this work, A.D. suggested a plan of deployment of the weapons basing on the criterion of the maximal probability of defeat of the attacking enemy. This criterion would be widely adopted afterwards.

A.D. stayed at the MAEA where he had worked as a lecturer at the department of higher mathematics for three years. In this period he communicated closely with academics from engineering departments, acquiring the corresponding, "applied", way of thinking.

**Teaching at the mechmath of the MSU, living in Riga and Minsk.** A year after graduating from the MAEA A.D. began to work part time at the department of differential equations of the mechmath. A number of well-known mathematicians such as V.V. Stepanov, who was the head of the department, V.V. Nemytsky, I.G. Petrovskii, and S.L. Sobolev also worked there. A.D. was responsible for practical training on ordinary differential equations and mathematical physics. Among his students were O.A. Ladyzhenskaya and O.A. Oleinik, subsequently outstanding women-mathematicians and academicians of the Soviet Academy of Sciences. This was the time when A.D. published his first scientific papers. Interestingly, the areas were very various: there were papers on ordinary differential equations, partial differential equations, stability theory etc. The candidate thesis written by A.D. under the supervision of I.G. Petrovskii and defended in June 1946 was devoted to the so-called modified Dirichlet problem for the Laplace equation in a general  $n$ -dimensional domain. During this period, A.D. published totally 10 papers and was awarded the prize of the Moscow Mathematical Society for his article in the journal "Soviet Mathematical Surveys."

Since then, the mechmath seminars on ordinary and partial differential equations had played a very significant role in the life of A.D. These seminars were led by such mathematicians as V.V. Stepanov, I.G. Petrovskii, A.N. Tikhonov, I.N. Vekua, L.A. Lyusternik, and later by O.A. Ladyzhenskaya, O.A. Oleinik, M.I. Vishik etc. A.D. recalled that the famous International Conference "Petrovskii Seminar" had originated apparently from the seminar on partial differential equations. A.D. mentions this period as one of the most vivid in his life.

Heavy teaching load at the MAEA forced A.D. to think about changing his place of work. Thus he accepted the proposal of the Air Forces Personnel

Department to take an academic position at the Aviation Engineering Military School in Riga in June 1947. Acting at the School as a director of the department of higher mathematics, A.D. also worked part time at the Latvian State University where he organized an educational scientific seminar followed the pattern of the MSU.

Working on an applied problem, A.D. familiarized himself with a differential equation containing the retarded argument. It became clear later that equations of this type emerged in control theory, biology, economics, medicine, etc., while had not been systematically studied before. The open country appeared in front of A.D., and the results, as he wrote, fell thick and fast.

The first article by A.D. devoted to delay differential equations was published in "Soviet Mathematical Surveys" in 1949. In December 1949, A.D. finished his doctoral thesis on this topic, making a scientific breakthrough. The corresponding monograph [1] was released in 1951. It was the first monograph in the field of functional differential equations to appear in the mathematical literature.

Later on, the theory of functional differential equations would be further developed in the works of many noted mathematicians such as N.N. Krasovskii, Yu.S. Osipov, Yu.A. Mitropolskii, L.E. Elsgolts, R. Bellman, K. Cooke, J. Hale, T. Kato and others. Hundreds of papers would have been published, and international conferences would be organized in this field every year.

Since February 1950, A.D. had worked full time at the Latvian State University (LSU) as a head of the department of general mathematics. Here, in particular, A.D. began studying the hyperbolic systems of one spatial variable. His educational scientific seminar served as an important form of communication with students and academic personnel. There appeared the first issues of the Proceedings of the fizmat of the LSU. For students, A.D. wrote the manual "How to get ready to research" and organized a competition in solving  $\varepsilon - \delta$  problems from the beginning of mathematical analysis. The brochure based on this material became subsequently a big success not only in Riga but even in the USA. The flat of A.D. in Riga was open to everyone for tutorials and scientific conversations. A.D. gradually became well-known in Latvia. However, the situation was overshadowed by a conflict between A.D. and the faculty administration. A.D. put the research at the departments in the first place while the Dean of the faculty considered public work of paramount importance. Therefore the administration hampered to nominate A.D. for professorship. There was also the family reason, as A.D.'s second son, Mitya, suffered from bronchial asthma and pneumonia

in the dewy climate of Riga.

Thus in autumn 1953 A.D. changed his job in the LSU to a position in the Byelorussian State University (BSU). But rooted in Riga A.D. had continued his collaboration with Lettish mathematicians long since then. During his Riga period A.D. published about 30 papers in the fields of boundary value problems in domains with complex boundaries, delay differential equations, potential theory, mixed problems for linear systems of partial differential equations, fixed points theorems for multivalued mappings etc.

A.D. worked in Minsk as a head of the department of differential equations of the BSU. He was the only doctor of science in mathematics in Minsk. A.D. described Byelorussians as balanced, open-hearted, and well-wishing people favourable for any kind of collaboration.

The educational and research seminar for students and postgraduates began its work in the lent term of 1953/54 under the direction of A.D., who supervised 8 postgraduate students from Riga and Minsk at that time. His own scientific interests were then connected with the investigation of mixed problems for hyperbolic systems of one spatial variable, the study of stationary points of a general autonomous system with switching on the plane etc. A.D. published 13 papers in Minsk, he also translated and edited the monograph "Stability theory of differential equations" by R. Bellman. By the end of his Minsk period, A.D. met such outstanding mathematicians as M.A. Krasnosel'skii, S.G. Krein, N.I. Ahiezer, and others. Later, these connections would play an important role in the life of A.D.

Unfortunately, despite of the good relations with students and academics from the BSU, A.D. had to leave Minsk as his family could not get a proper flat there. N.I. Ahiezer knew that the Kharkiv Aviation Institute (KhAI) had a good vacant flat that could have been immediately moved into. So N.I. Ahiezer helped A.D. to move to Kharkiv. A.D. had a clear view of the high scientific level in Kharkiv where N.I. Ahiezer worked with the other distinguished mathematicians B.Ya. Levin, V.A. Marchenko, and A.V. Pogorelov. Finally, Kharkiv was the home town for A.D. since A.D. spent his childhood there.

Therefore in September 1956 the second part of the life in Kharkiv started for A.D. after 24-year break. This period would last for 18 years.

**The Kharkiv period.** The family of A.D. moved to Kharkiv in autumn 1956, where A.D. headed the department of higher mathematics at the Kharkiv Aviation Institute. Within the next few years he had wrote his famous textbook on higher mathematics for technical colleges [2], which A.D. considered as one of his major contributions. The textbook was based on the proposition that the statement of higher mathematics for technical

colleges should be focused on applications. A.D. thought it was necessary to reduce the gap between the mathematics taught and the mathematics applied. These ideas were later developed in his lecture course [3], books [4, 5], and the collaboration with academician Ya.B. Zel'dovich.

Formation of "Physical-technical institute for low temperatures" of the Ukrainian Academy of Sciences (PTILT) in Kharkiv gave rise to a new area of research. A.D. had to organize and head the department of applied mathematics at the PTILT. Graduates from KhAI, the engineers with a mathematical bias, constituted the core of this department. Among them were such pupils of A.D. as V.G. Babskii, N.D. Kopachevskii, L.A. Slobozhanin, and A.D. Tyuptsov. All of them would later become well-known scientists.

Shortly after its foundation, the department began working on the fundamental problem of fluid oscillation in low gravity fields where the capillary forces and self-gravitation had to be taken into account. These investigations met the needs of the famous organization "Energija" led by S.P. Korolyov and interested in behaviour of liquid fuel in a rocket tank under the conditions close to zero-gravity. Soon enough, the following three directions of research became clear: problems of hydrostatics (nonlinear problems of equilibrium surfaces of capillary liquid, the stability of equilibrium states, their bifurcation, and stability margin); small oscillations of fluid approaching equilibrium with a free surface crooked by capillary forces; convection of weightless fluid caused by self-gravitation and the thermo-capillary effect. At that time the department of A.D. collaborated actively with the scientific schools of S.G. Krein in Voronezh and V.I. Yudovich in Rostov-on-Don, which was reflected in wide application of functional analysis to the problems of fluid mechanics. The study of these problems and related topics took quite a long time and resulted in the monograph on the low-gravity fluid mechanics [6], the first of its kind to appear in the mathematical literature. The monograph was written by A.D. in co-authorship with his four pupils. Its revised version was published in English in 1987 [7] and in Russian in 1990.

According to N.D. Kopachevskii, the supervision of A.D. was unobtrusive and very delicate, and created favourable conditions for fast scientific growth of his pupils. N.D. Kopachevskii recalls also that A.D. considered his work in the department of applied mathematics of FTILT as the most efficient and interesting in his scientific career. The cultural life in the department was very rich too. A.D. used to take the group to hiking tours. Some of those tours, in Crimean mountains and in Ala Tau, would always remain in the memory of his pupils.

Being a multi-aspect researcher A.D. did not restrict himself to studying

low-gravity fluid mechanics only. Parallel to this subject he had continued his work on the mathematical problems close to his mind. In particular, he published two surveys on the then current state of the theory of functional differential equations and had studied the mixed problem for semilinear systems of equations of hyperbolic type. There were also a few papers on impulse systems excited at prescribed times. Several articles were devoted to methodological questions concerning the essence of applied mathematics. A.D. translated and edited the interesting course on ordinary differential equations by F. Tricomi. In addition, the book "Elements of Applied Mathematics" in co-authorship with Ya.B. Zel'dovich came off the press in 1965 (the second edition of the book was already released in 1967). In collaboration with the colleagues from Voronezh (Yu.G. Borisovich and pupils) A.D. began studying multivalued mappings and differential inclusions.

Those years A.D. took an active part in the Committee on mathematical education under the Academy of Sciences of the USSR and in the Methodological Council in mathematics under the Ministry of Education of the USSR. Together with Ya.B. Zel'dovich, A.D. spoke in the press about the need of modernization of the curriculum in mathematics for secondary schools.

**Working at the MIIT.** In September 1974, A.D. took the position of a professor at the Moscow Institute of Transport Engineers (MIIT). He was finishing the book "Low-Gravity Fluid Mechanics" and writing papers on functional differential equations at that time.

Working at the department of applied mathematics of MIIT, A.D. organized a permanent seminar on differential equations and related topics. He participated actively in scientific conferences, both all-USSR and international. After the English edition (essentially revised and supplemented) of "Low-Gravity Fluid Mechanics" was out in 1987, A.D. switched to another subject and wrote "Elements of the theory of mathematical models" by 1991.

During the period from 1974 to 1991, A.D. published totally 93 scientific papers and was the author of seven books. The following topics lied in the range of his interests at that time: mechanics of capillary fluid, functional differential equations, variational and boundary value problems for elliptic partial differential equations, multivalued mappings, asymptotic and oscillation properties of operator differential equations, Volterra integral equations in a metric space with measure, extremum conditions in spectral isoperimetric problems with variable boundaries, the problem of rolling of a solid along two tracks (the last one arose in MIIT), the phenomena of stabilization and destabilization under small dissipative forces in nonconservative systems,

new solution properties in the problem of transverse vibration of a thread with beads, and others.

After 1991, A.D. periodically went on academic trips to the USA, Israel, Brasil etc., where he met his foreign colleagues. He had continued to work intensively in different directions as before.

The International conference on nonlinear analysis and functional differential equations "ADM-2000" dedicated to the 80-th anniversary of A.D. Myshkis took place in Voronezh in May 2000. From 2002 through 2008 A.D. visited regularly the famous Crimean Autumn Mathematical School-Symposium organized in the village of Laspi by N.D. Kopachevskii and the department of mathematical analysis of the Taurida National University. There A.D. lectured mathematics, debated methodological questions, and walked a lot in the environs. He was in a strong mathematical shape and enjoyed meeting his colleagues and pupils very much.

During his last years A.D. published more then 80 papers including the fundamental one on the mixed functional differential equations [8].

Summarize the activities of the eminent scientist and educator, a very kind and interesting man, A.D. Myshkis. He may be regarded by right as the founder of a number of scientific schools. He was the official supervisor for 36 candidates of sciences. Seven of them subsequently gained doctoral degrees. A.D. was the author and co-author of 17 books run into 43 editions in 10 languages and 332 scientific articles, translated and edited 16 books.

Unfortunately, A.D., as well as his outstanding friends M.A. Krasnosel'skii and S.G. Krein, had not become a member of the Academy of Sciences of the USSR (Russian Academy of Sciences) yet undoubtedly deserved it. The teacher of A.D. Myshkis, academician I.G. Petrovskii, regarded A.D. as one of his best pupils.

#### REFERENCES

- [1] Myshkis A.D. *Linear differential equations with retarded argument*. - Gostehizdat, 1951 [in Russian]; German transl.: Veb. Deutsch. Verl. Der Wiss., 1955.
- [2] Myshkis A.D. *Lectures on higher mathematics*. — M.: Nauka, 1964 [in Russian]; English transl.: Mir Publishing House, 1972.
- [3] Myshkis A.D. *Mathematics for students of higher technical institutions: special courses*. — M.: Nauka, 1971 [in Russian]; English transl.: Mir Publishing House, 1975.
- [4] Zel'dovich Ya.B., Myshkis A.D. *Elements of applied mathematics*. — M.: Nauka, 1965 [in Russian]; English transl.: Mir Publishing House, 1976.
- [5] Blekhman I.I., Myshkis A.D., Panovko Ya.G. *Applied mathematics: object, logic, and details of approaches*. — Kiev: Naukova Dumka, 1976 [in Russian]; German transl.: Veb. Deutsch. Verl. Der Wiss., 1984.



- [6] Babskii V.G., Kopachevskii N.D., Myshkis A.D., Slobozhanin L.A., Tyuptsov A.D. *Hydromechanics in zero gravity*. — M.: Nauka, 1976 [in Russian].
- [7] A. D. Myshkis, V.G. Babskii, N.D. Kopachevskii, L.A. Slobozhanin, A.D. Tyuptsov *Low-Gravity Fluid Mechanics. Mathematical theory of capillary phenomena*. — Berlin: Springer-Verlag, 1987.
- [8] A. D. Myshkis. Mixed functional differential equations // *Contemporary Mathematics. Fundamental Directions*. — M.: MAI Press, 4(2003), p. 5–120; English transl.: *Journal of Mathematical Sciences*, 129:5(2005), p. 4111–4226.